Optimization of Input Impedance and Mechanism of Noise Suppression in a DC SQUID RF Amplifier

M. Tarasov and Z. Ivanov

Abstract— **The optimal input impedance and noise of a dc SQUID RF amplifier at frequencies of the order of 1 GHz with a resonant input matching circuit has been studied analytically, numerically, and experimentally. A model for noise temperature and power gain has been developed for the practical resonant input tank circuit. A new effect of the output noise increasing or decreasing with changing the sign of voltage-to-flux transfer coefficient has been observed experimentally and explained analytically. The different values of noise temperature for the opposite $dV/d\Phi$ values have been interpreted using a model with partially correlated current and voltage noise sources. The equivalent layout for optimal input matching of a SQUID amplifier comprising series and parallel resonant circuits has been presented. Using such matching circuit and SIS junction as a signal source the SQUID amplifier noise temperature about 1 K has been measured at 1.1 GHz.**

I. Introduction

According to the estimations [1]–[8], dc SQUID RF amplifiers (SQA) could be the most sensitive low-noise amplifiers at frequencies up to several THz. At frequencies below 1 MHz, an energy sensitivity of the SQA close to the quantum limit has been achieved by several groups. In the 10–1000 MHz frequency range, a noise temperature of the order of 1 K has been reported in [1], [5]–[7]. It should be mentioned that at present only low-frequency applications below 1 MHz are well developed for biomedical and geophysical magnetometers and gradiometers, as well as for NMR measurements. For SQUID magnetometers, an impedance matching is usually not required and low-frequency devices are operated in SQUID-picovoltmeter (or SQUID-picoamperemeter) modes when the impedance of the signal source is much more (or much less) than the SQUID input impedance. At RF, it is usually necessary to measure not the amplitude of the small field, but its power and a perfect impedance matching is required. The lack of a practical SQA at higher frequencies is basically due to the not-so-clear understanding of the principles of SQA matching and less developed input matching circuits.

In this paper, we calculate the output noise, gain, and input noise temperature of SQA with different resonant matching circuits. We also compare with experimental results and explain the specific correlation-dependent mechanism of noise depression. The optimal input circuit layout has been calculated for the practical SQUID parameters.

Manuscript received October 11, 1995; revised March 8, 1996. This work was supported by the Swedish Royal Academy of Sciences, International Science Foundation, under Grant MOT000 and the Russian State Program on HTc Superconductivity under Grant 92009.

M. Tarasov is with the Institute of Radio Engineering and Electronics, Moscow 103907, Russia.

Z. Ivanov is with the Physics Department, Chalmers University of Technology, S-412 96 Gothenburg, Sweden.

Publisher Item Identifier S 1051-8223(96)04879-8.

The problem of SQA matching to the signal source is connected with contradictive requirements of SQUID parameters. For higher gain and lower noise temperature T_n, the loop inductance should be decreased, but for perfect matching and good coupling to the input coil, it should be increased. The best performance can be achieved with loop inductance $L < 100$ pH. The input coil should have low capacitance to the SQUID loop in order to operate at high frequencies; that limits the turn ratio of input coil to the SQUID loop to about 4–10. As a result we have the optimized input coil inductance of about 1 nH which at frequencies below 1 GHz has an impedance below 6 Ω. This value should be compared with the resistance of the SQUID transformed into the input circuit. For a parallel equivalent circuit it is over 1 kΩ, and for a series circuit it is below 0.01 Ω. It means that the input resistance is shunted by an over 100 times as low impedance, or connected in series with 100 times as much impedance. The usual way to get rid of reactance is to arrange parallel or series resonant circuits, in which the reactance of the input coil will be compensated at the resonant frequency.

The next question is what real resistance should the signal source have to provide the lowest noise temperature. In general, when independent input current and voltage noise sources exist, the optimal input source required to achieve the uncertainty principle limit in energy sensitivity of a voltmeter is $R = (E_n^2/I_n^2)^{0.5}$, where E_n and I_n are the voltage and current noise amplitudes. But the expression does not take into account the power mismatch due to the difference between this noise-optimized resistance and the input resistance of the amplifier. The problem becomes more complicated when the correlation between current and voltage noise sources is taken into account.

Optimization of the SQA at radio frequencies in [1], [4], and [8] has been analytically studied for the case of a series input resonant tank circuit. This type of resonant circuit was chosen mainly due to the usual low-impedance source of signal, such as a superconducting input coil. Another reason is that a series input circuit allows to develop a simple analytical model. For such a circuit it was derived in [1] that the optimized noise temperature and signal source resistance are

$$T_n = \gamma_v \frac{\omega L}{\alpha^2 R} T \left[\frac{R_i^{\rm opt}}{\omega L_i} + \frac{\alpha^2 \omega L}{4 R_d} \right] \quad (1)$$

$$R_i^{\rm opt} = \alpha^2 \omega L_i \left[\left(\frac{\omega L}{4R_d} \right)^2 + \frac{\gamma_V \gamma_J - \gamma_{VJ}^2}{\gamma_V^2} \right]^{0.5} \approx \alpha^2 \omega L_i \tag{2}$$

where T_n is noise temperature, ω is signal frequency, T is bath temperature, R is normal resistance, R_d is dynamic resistance, L is SQUID loop inductance, L_i is input coil inductance, $M^2 = \alpha^2 L L_i$ is mutual inductance, α is coupling factor, and γ's are coefficients. Such an optimal input resistance $R_i^{\rm opt}$ is rather connected to the reactance of the input coil than to the input resistance. This is different from the usual idea of optimal impedance matching when reactances are tuned out and the real part of the signal source resistance equals the input resistance of the amplifier.

The following SQA noise temperature calculations have been done taking into account the input impedance matching, correlation of noise components in SQUID and mutual influence of input and output circuits of SQUID amplifier.

II. Transformation of SQUID Impedance into the Resonant Input Circuit

The dc SQUID with tightly coupled input coil in the first approximation can be viewed as a simple transformer. We can present the input impedance of bare SQUID in two equivalent models: parallel and series. In the former, an input coil inductance is connected in parallel with resistance transformed from the SQUID loop. This corresponds to the conversion of two Josephson junction dynamic resistances connected in series for the circular current (that is $4R_d$ of SQUID measured for the parallel connection for the output voltage) via the resistance transforming ratio

$$n^2 = \left(\frac{w_1}{w_2} \right)^2 = \frac{L_i}{\alpha^2 L}$$

as in usual voltage transformer, this brings parallel resistance in the input coil:

$$R^* = 4R_d n^2 = \frac{4R_d L_i}{\alpha^2 L}. \tag{3}$$

In the series model, the input circuit can be presented as a series connected input coil inductance and series resistance. We can recalculate the above value into the series circuit presenting the impedance of parallel circuit:

$$Z = \frac{i\omega L_i \cdot 4R_d n^2}{i\omega L_i + 4R_d n^2} = i\omega L_i + R^{**}$$

where R^{**} is series equivalent resistance. This brings

$$R^{**} = \frac{\alpha^2 \omega^2 L L_i}{4R_d + i\alpha^2 \omega L}$$

and if $\alpha^2 \omega L$ is small this brings the usual value of series resistance transformed into the series tank circuit

$$R^{**} \cong \frac{\omega^2 M^2}{4R_d} = \frac{4R_d n^2}{Q^2} \tag{4}$$

in which $M^2 = \alpha^2 L L_i$ is the mutual inductance of SQUID loop and input coil.

These resistances can be presented via Q-factor of the unloaded tank circuit, that should be the same for both cases. This brings $R^{**} = R^*/(1 + Q^2)$, in which $Q = R^*/(\omega L) = \omega L / R^{**}$.

III. Optimization of the SQA Output Noise Temperature

Noise at the SQA output, according to [1], consists of two main parts. The first is usual voltage noise with spectral density $S_v(f) = 4\gamma_v kTR$, where R is SQUID normal resistance and γ_v is voltage noise spectral coefficient, and the second part is due to the noise current J_n circulating in the SQUID loop and linked to the output via the input circuit. The equivalent noise voltage induced in the input circuit $E_n(t) = M\, dJ_n(t)/dt$, or $E(\omega) = -i\omega M J(\omega)$. The voltage in the input circuit produces current in the input circuit $I_1 = E_n(t)/Z$, where Z is total impedance of input circuit, and this current via usual SQUID transfer impedance $dV/dI_i = MV_\Phi$, where $V_\Phi = dV/d\Phi$, produces the output voltage

$$V_n = MV_\Phi I_1 = \frac{M^2 V_\Phi}{Z} \cdot \frac{dI_n}{dt} = \frac{MV_\Phi}{Z} \cdot \frac{dE_n}{dt}.$$

The noise square voltage at the output can be calculated as

$$V_{n,\,tot}^2 = \left(V_n + \frac{MV_\Phi}{Z} \cdot \frac{dE_n}{dt} \right)^2 = V_n^2 + 2V_n \frac{dE_n}{dt} \cdot \frac{MV_\Phi}{Z} + \frac{(MV_\Phi)^2}{Z^2} \left(\frac{dE_n}{dt} \right)^2$$

which can be expressed in spectral densities

$$4kTR\gamma_{\rm tot} = 4kTR\gamma_v + 2MV_\Phi \overline{V_n(t) \cdot \frac{dE_n}{dt}} + \left(\frac{MV_\Phi}{Z} \right)^2 \cdot M^2 \omega^2 \cdot \gamma_J kT$$

in which $\gamma_{\rm tot}$ represents the ratio of total noise at the output to the Johnson noise at the output resistance at bath temperature. Taking into account that

$$S_{VJ}(f) = \gamma_{VJ} kT = \frac{\overline{J_n V_n}}{\Delta f}$$

$$\overline{V_n \frac{dE_n}{dt}} = \overline{V_n \frac{{\rm Im}\, Z}{(|Z|)^2} M\omega \cdot J_n} = \frac{M\omega\, {\rm Im}\,(Z)}{(|Z|)^2} \overline{J_n V_n}.$$

The total impedance in the simple series tank circuit is

$$Z_s = i\omega L_i + R^{**} + (i\omega C + R_i^{-1})^{-1}$$

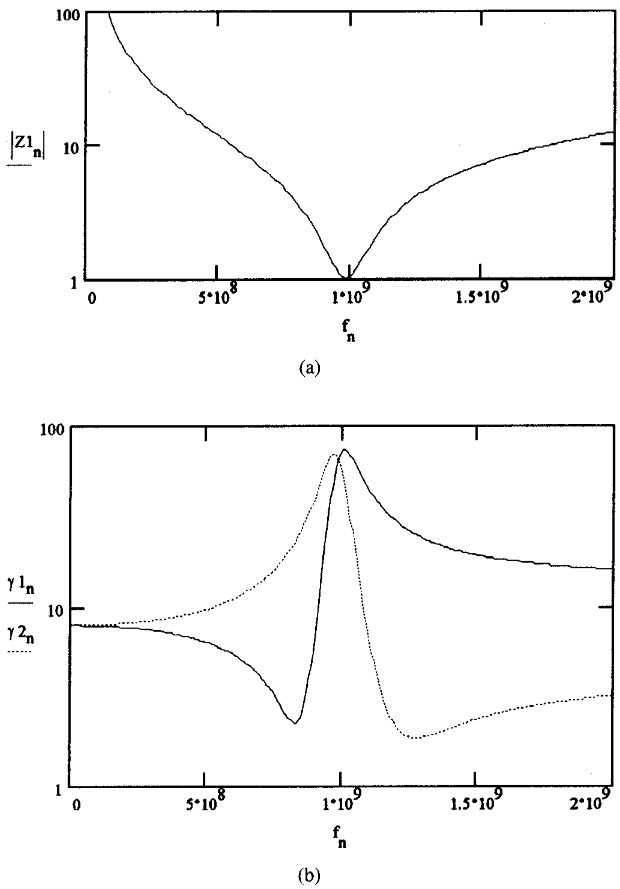

Fig. 1. (a) Series impedance $|Z1_n|$ frequency dependence calculated for tank circuit with real load; (b) dependencies of output noise $\gamma 1$ calculated for $V_\Phi = +R/L$ and $\gamma 2$ calculated for $V_\Phi = -R/L$ on frequency for the typical SQUID parameters: input resistance $R_i = 1\,\Omega$, L_i =1.3 nH, L =100 pH, C =20 pF, $R = 10\,\Omega$; $R_d = 100\,\Omega$.

and in parallel

$$Z_p = i\omega L_i + [i\omega C + (R^*)^{-1} + (R_i)^{-1}]^{-1}$$

and we can obtain in the case of pure real signal source impedance

$$\gamma_{\rm tot} \cong \gamma_v + 2M^2 V_\Phi \omega \frac{\omega L_i + \dfrac{1}{\omega C_i}}{(|Z|)^2} \cdot \frac{\gamma_{vi}}{R} + \left(\frac{M^2\omega V_\Phi}{R\cdot|Z|}\right)^2 \gamma_J.$$

The frequency dependencies of total input impedance $Z1$ and total γ-factor are presented in Fig. 1. One can see, that at resonant frequency we have over one order of magnitude increase of noise. Another important feature is sharp decrease of output noise of SQA at sidebands near the resonant frequency. The minimum of noise is observed at higher or lower sideband, depending on the sign of voltage-to-flux transfer coefficient. Such behavior cannot be interpreted by simple ideas of impedance matching without regard for the correlation between the noise components. Similar consideration in [1], without taking into account the sign of V_Φ function, does not allow to calculate two noise minima, below and above the resonant frequency.

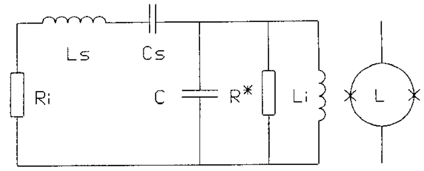

Fig. 2. Schematic layout of input resonant matching circuit. R_i is the signal source resistance, L_s is the series inductance, and C_s and C_p are the series and parallel additional capacitances, respectively.

IV. Input Noise Temperature

To obtain the noise temperature of SQUID voltmeter we apply at the input the signal with noise temperature T_n that will produce at the output the noise voltage square

$$V_s^2 = 4kT_nR_i \cdot \Delta f\left(\frac{MV_\Phi}{Z_t}\right)^2$$

equal to the noise of the SQA $4k\Delta f R_i\,[MV_\Phi/(Z_t)]^2 T_n = 4k\Delta f T R \gamma_{\rm tot}$:

$$T_n = T\cdot\frac{R}{R_i}\cdot\frac{Z^2}{M^2V_\Phi^2} \cdot\left[\gamma_v + \frac{2\gamma_{vi}M^2\omega V_\Phi}{R\,(|Z|)^2}\,{\rm Im}\,Z + \left(\frac{M^2\omega V_\Phi}{R|Z|}\right)^2\gamma_J\right]. \quad (5)$$

In the resonant case ${\rm Im}\,Z = 0$, the expression is simplified to

$$T_n = \frac{RT}{R_iM^2V_\Phi^2}\left[\gamma_v\,(|Z_1|)^2 + \left(\frac{M^2\omega V_\Phi}{R}\right)^2\gamma_J\right].$$

The optimal R_i can be obtained by

$$\frac{dT_n}{dR_i} = 0$$

which in the resonant case

$$Z_1 = R_i + \frac{4RL_i}{\alpha^2 L}$$

for parallel, or

$$R_i + \frac{\omega^2M^2}{4R_d}$$

for series circuit, brings a simple relation for the parallel tank circuit:

$$(R_i^{paral})^2 = (R^*)^2 + \alpha^4\,\frac{\gamma_J}{\gamma_v}\,(\omega L_i)^2$$

and

$$(R_i^{ser})^2 = (R^{**})^2 + \frac{\gamma_J}{\gamma_V}\,(\alpha^2\omega L_i)^2$$

which in the usual case $\omega L_i \ll R^*$ brings $R_i^{\rm opt} = R^*$ that can be clearly explained as a perfect resistive matching of the output load via usual transformer. But the value of T_n in this case can be high due to the large γ_1 and γ_2 (see Fig. 1) which are of the order of 100. The origin for that is the last term in (5), which increases as $|Z_1|^{-2}$ contrary to $|Z_1|^{-1}$ for the second one. To reduce the γ-factor at signal frequency the Z_1 should be increased.

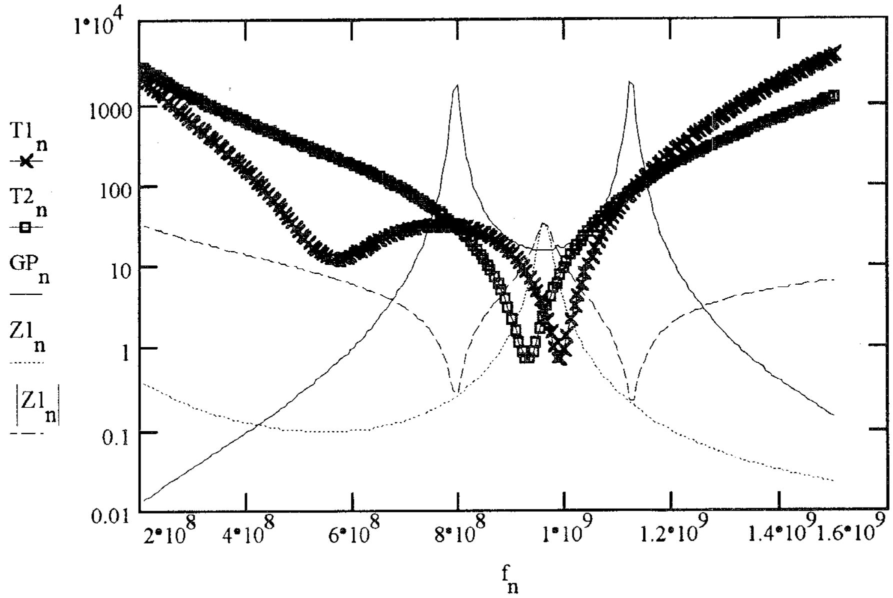

Fig. 3. Frequency dependencies of input noise temperature $T1$, $T2$ calculated for two opposite values of $V_\Phi = R/L$, power gain GP, and input tank circuit impedance real part $Z1$ and module $|Z1|$ for $R_i = 2\,\Omega$, $C_s = 2\,\text{pF}$, $L_s = 12\,\text{nH}$, $C = 20\,\text{pF}$, $L_i = 1.3\,\text{nH}$, $L = 100\,\text{pH}$.

Fig. 1 brings clear impression that to obtain low noise operation of SQA it is necessary to design a specific matching circuit that increase the real part of input resistance $Z1$ at the signal frequency. It is impossible by simple series resonant circuit, but can be achieved by series-parallel connection of signal source (see Fig. 2). For such circuit, we calculated the total impedance Z_1 of the input circuit, matching, and available power gain G_p of the amplifier, and the input noise temperature $T_n = T_{out}/G_p$. The results are presented in Fig. 3.

One can see that in some frequency range, we have maximum of impedance which brings the noise temperature below 1 K and the gain is not so high as at noise maxima, where it is over 1000, but it is still high (over 30). Tracks for $Z1$ and $|Z1|$ shows strong correlation between this impedance and both power gain and noise temperature. It should be mentioned that in numerical calculations of γ factors, the reactance of series L_s-C_s circuit should be accounted instead of simple $\omega L - 1/\omega C$ relation for the imaginary part of impedance.

V. Mechanism of Noise Reduction in SQA

The second term in (5) depends on resonant term and V_Φ, the latter has usually sinusoidal dependence on the input magnetic flux. This term can change its sign both dependent on input flux and frequency. It can be negative in some frequency range over the resonant frequency when $V_\Phi > 0$ and in some frequency range below the resonant frequency when $V_\Phi < 0$. The maximum in such noise reduction can be obtained by setting $dT_n/d\omega = 0$ which gives a very simple relation

$$\left(\omega_x L_i - \frac{1}{\omega_x C_i}\right)^2 = (R_i + R^{**})^2$$

and now if we take the transformed value (4) of R_i we can get

$$\omega_x L_i - \frac{1}{\omega_x C_i} = \pm\left[\alpha^2 \omega L_i \frac{(\gamma_V \gamma_J - \gamma_{VJ}^2)^{0.5}}{\gamma_v} + \frac{\omega_x^2 M^2}{4R_d}\right]$$

in the right side first term dominates and we can get noise-depression frequency center

$$\omega_x^{-2} = L_i C_i \left[1 \pm \alpha^2 \frac{(\gamma_V \gamma_J - \gamma_{VJ}^2)^{0.5}}{\gamma_V}\right]$$

which for typical experimental parameters $\gamma_V = 8$, $\gamma_J = 1.1$, $\gamma_{VJ} = 3$, $\alpha^2 = 0.8$ brings $\omega_+ \approx 1.1\omega_0$ and $\omega_- \approx 0.92\omega_0$, where $\omega_0 = (L_i C_i)^{-0.5}$.

Now we can derive the relation for the ΔT_n in these specific points just by substituting ω_x:

$$\begin{aligned}\Delta T_n(\omega_x) &\cong \pm T \frac{\omega^2 M^2}{R R_i} \gamma_{VJ} \frac{(\gamma_V \gamma_J - \gamma_{VJ}^2)^{0.5}}{\gamma_V} \\ &\cong \pm T \frac{\omega L}{R} \gamma_{VJ}.\end{aligned}$$

We can estimate also the uncompensated reactance

$$Z \cong \alpha^2 \sqrt{\frac{L_i}{C_i} \cdot \frac{\gamma_V \gamma_J - \gamma_{VJ}^2}{\gamma_V^2}}.$$

Applying the usual ideas of impedance matching to such case, we can obtain the available power matching ratio $K_p = P_{\text{load}}/P_{\text{source}}$:

$$\begin{aligned}K_p &= \frac{4R_i \operatorname{Re} Z_i}{|R_i + Z_i|^2} \\ &\cong \alpha^2 \frac{\omega L}{R_d}\left(1 + \alpha^2 \frac{\gamma_V \gamma_J - \gamma_{VJ}^2}{\gamma_V^2}\right)^{-1}\end{aligned}$$

and this is of the order of 0.1 for usual parameters. This is a fundamental limit that reduces the available power gain in the low-noise case.

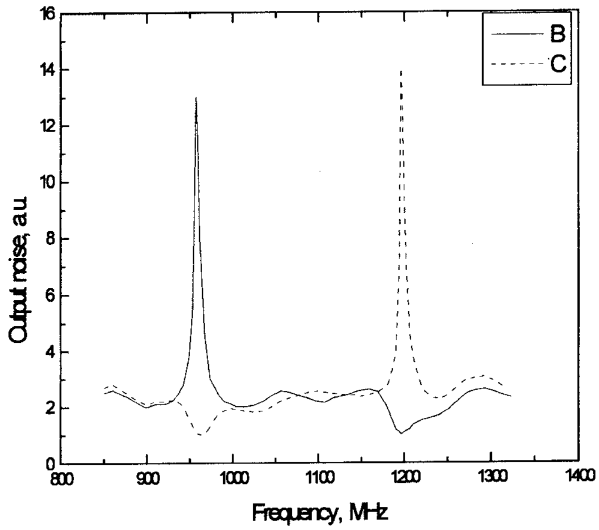

Fig. 4. The output noise of SQUID amplifier measured with load comprising 0.3 Ω SIS junction in series with 3.7-pF capacitor. The Q-factors are over 200 at the first and below 200 at the second maximum, which corresponds to transformed resistance R^* about 1 kΩ and 2 kΩ.

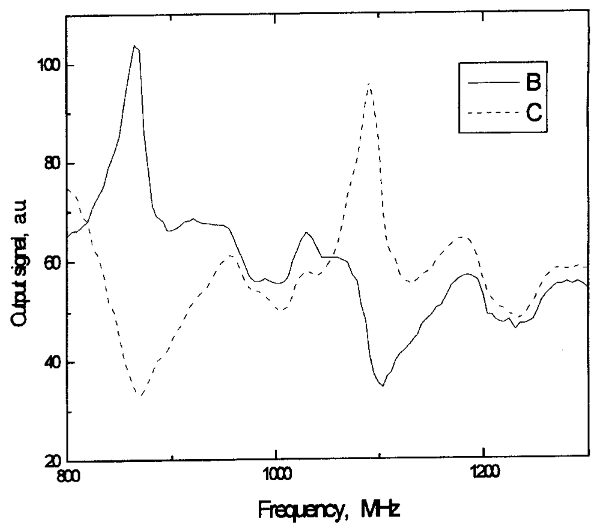

Fig. 5. Frequency dependencies B and C of the SQA output noise measured for two levels of input magnetic flux. The input of SQA is loaded by 36 Ω SIS junction in series with about 5 Ω inductance. The resonant maxima are at 838 and 1078 MHz and minima at 858 and 1100 MHz.

VI. Comparison with Experiments

The experiments were performed with dc SQUID's described in [10] with input circuit similar to presented in Fig. 2. In the case when a low input resistance is connected at SQA input in series with small capacitor, the influence of the input circuit on the output noise is clearly observed. In Fig. 4, two output noise frequency dependencies are presented. For both of them the input magnetic flux tuning was performed so that output noise at resonant frequency is maximum. The first maximum at 944 MHz is obtained when Φ_e tuning tends to increase the critical current, and in the second case at 1236 MHz when the sign of $dI_c/d\Phi_e$ is opposite. When the sign of $dV/d\Phi$ is changed, it leads to about an order of magnitude increase of noise in one frequency mode and decrease in another, as it is shown in Fig. 1(b). Q-factors for both cases corresponds to unloaded circuit and are determined by the SQUID resistance transformed into the input circuit. Another value of Q-factor can be observed at Fig. 5, that corresponds to 36Ω signal source resistance. Here Q-factor is determined by external load. These experimental results illustrate the dependence of the SQA output noise on the $dV/d\Phi$ sign and can be clear explained compared to the calculated frequency dependence of output noise, input noise temperature and gain, see Figs. 1 and 3.

The resonant features of the input circuit corresponds to the values of circuit elements: parallel capacitance 20 pF, input series inductance of cable and connectors 5 nH, input series capacitor 4 pF, input coil inductance 1.3 nH, signal source resistance 0.3 Ω or 36 Ω, SQUID dynamic resistance 130 Ω.

Comparison with calculated optimal gain and noise in Fig. 3 leads to the conclusion that to improve the SQA performance it is necessary to reduce the distance between the source of signal and SQA, that will reduce inductance and improve Q-factor, noise temperature, and gain. For better matching in present configuration a $\lambda/4$ microstrip transformer can be used.

Using optimized series-parallel matching circuit and SIS junction as a source of calibrated shot noise the SQUID amplifier noise temperature $T_n \cong 1$ K and power gain $G \cong$ 10 dB have been measured at signal frequency 1.1 GHz in 50 MHz bandwidth [11].

VII. Conclusion

The noise temperature and optimal signal source impedance have been studied in detail both analytically and numerically for SQUID amplifier with different resonant input circuits. A new effect of the output noise increasing or decreasing with changing the sign of voltage-to-flux transfer coefficient has been observed experimentally and explained analytically and numerically. A simple mechanism of noise reduction in SQUID amplifier due to noise correlation has been presented. It was shown that it is impossible to obtain low noise temperature at resonant frequency of single resonant tank circuit due to the rise of circulating current noise impact. It is possible to tune to the sidebands below or above the resonant frequency, but in such case a pronounced mismatch appears due to uncompensated reactance. For practical amplifiers, it can be helpful to use double resonant circuit with parallel and series resonant modes and a $\lambda/4$ coaxial transformer to achieve the signal source optimal resistance. The noise temperature (about 1 K) has been obtained at 1.1 GHz for integrated multiloop SQUID amplifier with optimized series-parallel input resonant circuit.

Acknowledgment

The helpful discussions with T. Claeson and V. Koshelets are greatly acknowledged.

REFERENCES

[1] J. Clarke, C. Tesche, R. P. Giffard, and J. Low, *Temp. Phys.*, vol. 37, no. 3/4, pp. 405–420, 1979.
[2] R. F. Voss, *Appl. Phys. Lett.*, vol. 38, no. 3, pp. 182–184, 1981.
[3] R. H. Koch, D. J. Van Harlingen, and J. Clarke, *Appl. Phys. Lett.*, vol. 38, no. 5, pp. 380–382, 1981.
[4] C. D. Tesche, *Appl. Phys. Lett.*, vol. 41, no. 5, pp. 490–492, 1982.
[5] D. G. McDonald, *Appl. Phys. Lett.*, vol. 44, no. 5, pp. 556–558, 1984.
[6] C. Hilbert and J. Clarke, *Appl. Phys. Lett.*, vol. 45, no. 7, pp. 799–801, 1984.
[7] ——, *IEEE Trans. Magn.*, vol. MAG-21, no. 2, pp. 1029–1031, 1985.
[8] C. Hilbert, J. Clarke, and J. Low *Temp. Phys.*, vol. 61, no. 3/4, pp. 263–280, 1985.
[9] T. Takami, T. Noguchi, and K. Hamanaka, *IEEE Trans. Magn.*, vol. 25, no. 2, pp. 1030–1033, 1989.
[10] M. Tarasov, V. Belitsky, and G. Prokopenko, *IEEE Trans. Appl. Superconduct.* vol. 2, pp. 79–83, 1992.
[11] M. Tarasov, L. Filippenko, A. Baryshev, and A. Vystavkin, Th. de Graauw, W. Luinge, in *Proc. EUCAS-1995,* Edinburgh, Scotland, July 3–6, 1995, pp. 763–768.

M. Tarasov, photograph and biography not available at the time of publication.

Z. Ivanov, photograph and biography not available at the time of publication.