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Radiation linewidth of flux-flow oscillators

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Abstract
The results of linewidth measurements on flux-flow oscillators (FFOs) of a new design with improved parameters are presented. Extensive measurements of the dependence of the free-running FFO linewidth on the differential resistances associated both with the bias current and the control-line current (applied magnetic field) are taken. A profile of the FFO radiation line is measured in different regimes of FFO operation and compared to the theoretical models. A Lorentzian shape of the FFO line is observed both at Fiske steps (FSs) in the resonant regime and on the flux-flow step (FFS) at high voltages. A phenomenological model of the FFO linewidth taking into account all known noise sources (both internal and external) is used to explain the FFO linewidth dependence on the experimental parameters. Finally, we discuss the feasibility of using an electronic phase-locking loop (PLL) over the entire FFO operational frequency band.

1. Introduction

Flux-flow oscillators (FFOs) [1] are presently being developed for integration with SIS-mixers for use in submillimeter-wave all-superconductor integrated receivers (SIRs) [2]. The FFO is a *long* Josephson tunnel junction in which an applied dc magnetic field and a dc bias current, I_B, drive a unidirectional flow of fluxons, each containing one magnetic flux quantum, $\Phi_0 = h/2e \approx 2 \times 10^{-15}$ Wb. Symbol h is the Planck's constant and e is the elementary charge. An external coil or an integrated control line with current, I_{CL}, can be used to generate the dc magnetic field applied to the FFO. According to the Josephson relation the junction biased at voltage V oscillates with a frequency $f = (1/\Phi_0)^*V$ (about 483.6 GHz mV^{-1}). The velocity and density of the fluxons and thus the power and frequency of the emitted mm-wave signal may be adjusted independently by both the bias current and the magnetic field. A noise temperature below 100 K (DSB) was achieved at 500 GHz for a single chip SIR [2].

Recently, the intrinsic FFO linewidth has been measured in the frequency range of 250–650 GHz [3]. The feasibility of phase locking the FFO to an external reference oscillator was demonstrated experimentally and an FFO linewidth as low as 1 Hz was measured in the frequency range of 270–440 GHz [4] on steep Fiske steps (FSs), where the intrinsic FFO linewidth is about 1 MHz. The intrinsic linewidth increases considerably at voltages above a boundary voltage of the order of 1/3 of the superconducting gap voltage as a result of the abrupt increase in the internal damping due to Josephson self-coupling [5]. In this paper the results of extensive measurements of the radiation linewidth as well as a profile of the FFO emission are presented and compared to the theoretical models.

2. FFO radiation linewidth

Presently, no commonly accepted theory exists for the FFO linewidth, but preliminary estimations can be made [6] on the basis of the general theory for the radiation linewidth of the *lumped* Josephson tunnel junction [7, 8]. The linewidth,

δf, of a small Josephson junction is mainly determined by low frequency current fluctuations. For white noise it can be written (see e.g. [8]) as

$$\delta f = \left(2\pi/\Phi_0^2\right)\left(R_d^B\right)^2 S_i(0) \qquad (1)$$

where $S_i(0)$ is the power density of the low frequency current fluctuations, $R_d^B = \partial V/\partial I_B$ is the dc differential resistance which transforms current fluctuations to voltage (and phase) noise:

$$S_i(0) = (e/2\pi)\{I_{\rm qp}\coth(v) + 2I_s\coth(2v)\}$$
$$\text{with} \quad v = (eV_{\rm dc})/(2k_{\rm B}T_{\rm eff}) \qquad (2)$$

where $k_{\rm B}$ is Boltzmann's constant. $I_{\rm qp}$, I_s and $V_{\rm dc}$ are, respectively, the quasiparticle current, the superconducting current and the averaged dc voltage in the bias point. $T_{\rm eff}$ is the effective temperature of the quasiparticles in the junction electrodes. This formula describes a nonlinear superposition of thermal and shot noise. The supercurrent itself cannot be a source of fluctuations because of its reactive character; the additional term proportional to I_S appears due to the interaction of the supercurrent with the embedding circuit. It should be noted that formula does not take into account the spatial variation of the tunnel current along the FFO, the interactions of the moving fluxons and the influence of the external low frequency interference. All these effects are believed to increase the FFO linewidth. Nevertheless, very recently it was shown [9] that a formula similar to equation (2) could be applied for a distributed Josephson junction in the limit where there are many fluxons in the junction.

Fluctuations in the external magnetic field (which affect the average distance between fluxons) can be accounted for by the differential tuning resistance of the control line $R_d^{\rm CL} = \partial V_{\rm FFO}/\partial I_{\rm CL}$ for fixed dc bias current I_B. In the case of an *external interference* besides R_d^B also $R_d^{\rm CL}$ 'converts' low frequency external noise currents, $I_{lf}^{(B,{\rm CL})}$, to frequency fluctuations following the same relations:

$$\delta f \propto R_d^{(B,{\rm CL})} * I_{lf}^{(B,{\rm CL})}. \qquad (3)$$

A numerical model of the FFO based on equations (1–3) and taking into account the thermal noise from the biasing resistors, as well as external low frequency interference via the bias and control line circuitry has been developed [6]. It was shown [6] that the thermal noise in the biasing resistors is negligibly small in a well-designed bias and control line circuitry. The radiation linewidth for different models of the noise contributions calculated using measured FFO parameters are presented in figure 1 as a function of the differential resistance R_d^B. Curve 1 is calculated using the standard tunnel junction model [7, 8], equations (1) and (2). In this model δf is proportional to $\left(R_d^B\right)^2$. Curve 2 is calculated taking into account only the low-frequency external fluctuations via the bias current leads, equation (3), $I_{lf} = 0.1\ \mu$A. Curves 3 and 4 include both white noise and external fluctuations. The external fluctuations via both bias and control-line current leads are calculated for two different $R_d^{\rm CL}$ values: $R_d^{\rm CL}l = 0.002\ \Omega$ typical for the resonant regime on the Fiske step (FS) (curve 3), and $R_d^{\rm CL}2 = 0.1\ \Omega$ in the high-damping flux-flow regime for $V > V_{\rm JSC}$ (curve 4). One can see that

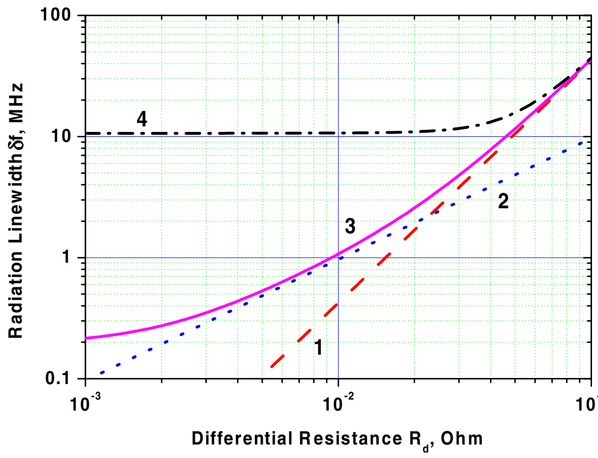

Figure 1. Calculated dependence of the radiation linewidth δf on the differential resistances R_d for typical FFO parameters: $T_{\rm eff} = 4.2$ K, $V_{\rm dc} = 1$ mV, $I_{\rm qp} = 2.5$ mA, $I_s = 7.5$ mA, $I_{lf} = 0.1\ \mu$A (see text for details).

the FFO linewidth at $R_d^B < 0.01\ \Omega$ is mainly determined by fluctuations coming from the control line. The results presented in figure 1 demonstrate the importance of the $R_d^{\rm CL}$ value—even in the simple FFO model for quite low levels of the external low frequency fluctuations ($I_{lf} = 0.1\ \mu$A) the radiation linewidth is as large as 10 MHz in the flux-flow regime for $V > V_{\rm JSC}$.

3. FFO radiation profile

In order to avoid the resonant fine-structure reported in [10] for an FFO with standard rectangular overlap geometry, we have developed a new design of the FFO [11]. In this design the FFO is tapered from both sides in such a way that its width is decreased from 6 μm to 1.5 μm over a distance of 20 μm. As a result of this modification the resonant structure was almost totally suppressed and the FFO frequency can be continuously adjusted. It should be noted that a tapered FFO has larger output impedance than a standard FFO of rectangular shape. This considerably simplifies the matching to the following microwave circuits.

The absence of the resonance structure enables one to provide a detailed analysis of the free-running FFO spectrum, which has been measured using the integrated harmonic mixer technique [3]. A frequency detector system with a relatively low loop gain has been used to stabilize (frequency lock) the FFO. In this case it is assumed that only very low-frequency noise and drift are eliminated by the narrow-band feedback. Thus the linewidth, determined by much faster fluctuations, which are assumed to be the 'natural' ones, can be carefully measured. The shape of the FFO spectrum provides us with important information about the relationship between internal and external fluctuations as well as the spectral distribution of these fluctuations. According to the theory [8, 12] the shape is Lorentzian for wide-band fluctuations, whereas the profile is Gaussian for narrow-band electromagnetic ('technical') interference (EMI) with frequencies smaller than the autonomous FFO linewidth $\delta f_{\rm AUT}$.

The measurements on the flux-flow step (FFS) [11] demonstrate an excellent coincidence between the calculated

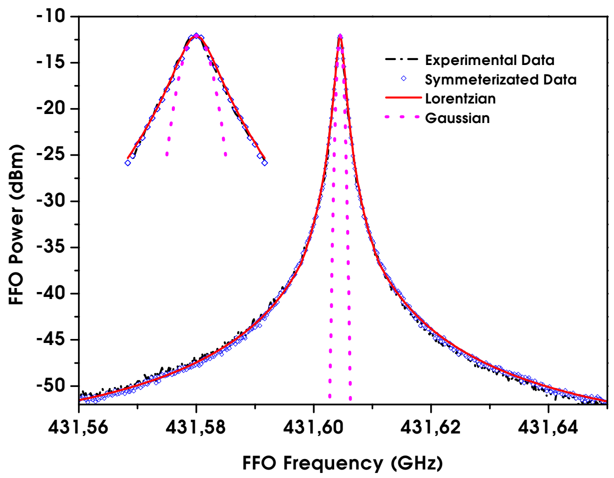

Figure 2. FFO spectrum measured when biased on the Fiske step ($V_{\mathrm{FFO}} = 893\ \mu$V, $R_d^B = 0.0033\ \Omega$, $R_d^{\mathrm{CL}} = 0.00422\ \Omega$, $\delta f_{\mathrm{AUT}} = 1.2$ MHz)—dash-dotted line. Diamonds show the symmetrized experimental data. Fitted theoretical Lorentzian and Gaussian profiles are shown by solid and dotted lines, respectively. The inset shows a zoom-in on the central peak with the frequency axis multiplied five times.

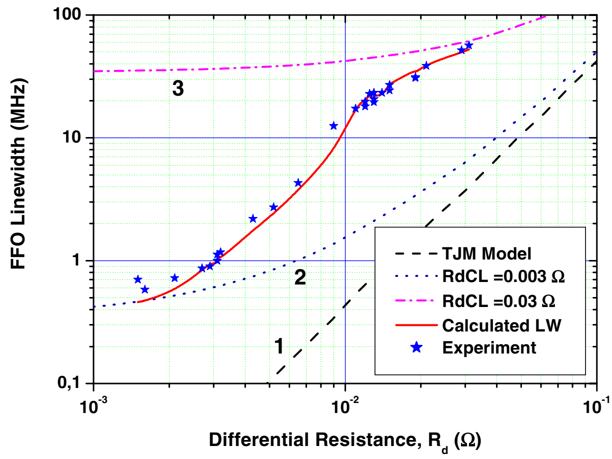

Figure 3. Dependence of the FFO radiation linewidth on the differential resistance R_d (see text). Curves 1–3 are calculated for the following experimental parameters: $V_{\mathrm{dc}} = 1$ mV, $I_{\mathrm{qp}} = 3$ mA, $I_s = 7$ mA, $T_{\mathrm{eff}} = 4.2$ K.

Lorentzian curve and the symmetrizated experimental data. This agrees with theoretical expectations [8, 12] for the pure flux-flow regime on the FFS with large differential (bias current) resistance R_d^B [5, 6]. The profile of the FFO line recorded when biased at the steep FS, where the differential resistance is extremely small, can be different from the one measured on the smooth FFS. According to the model [6] the linewidth at low R_d^B values is determined by external fluctuations coming mainly through the control line (see figure 1). The FFO spectrum measured at the FS is presented in figure 2; the theoretical curves are also shown in figure 2 for comparison. The theoretical lines providing the best fit near the peak are shown by the solid line and the dashed line for Lorentzian and Gaussian, respectively. The FFO line shape is very well approximated by the Lorentzian expression. For some experimental conditions a small deviation—so-called 'wings'—has been recorded, which can be accounted for by adding a second 'broader' Lorentzian component. Such line shape with 'wings' is typical [12] for an oscillator affected by both frequency and amplitude fluctuations. Furthermore, according to [12], the minor asymmetry of the measured FFO line profile can be ascribed to correlations between frequency and amplitude fluctuations, which is to be expected at the steep FS.

The data in figure 2 demonstrate that even at extremely low differential resistance R_d^B the emission line profile is Lorentzian; even at the "plateau" of curve 3 in figure 1 at $R_d^B < 0.01\ \Omega$. This fact is very important for linewidth calculations. It gives us experimental evidence that the wide-band fluctuations are present in the magnetic field, because according to [6] at small R_d^B the FFO linewidth is mainly determined by fluctuations of the magnetic field. Since an influence of the wide-band fluctuations in the biasing resistors feeding the control line (CL) is negligibly small [6], it means that there must be a channel for transfer of the wide-band fluctuations from the bias current I_B to the FFO via the magnetic field. Partially, it can be associated with a self-field effect (part of the bias current is flowing along the FFO electrodes creating a magnetic field proportional to I_B). Under these circumstances the role of the differential resistance of the CL, $R_d^{\mathrm{CL}} = \partial V_{\mathrm{FFO}}/\partial I_{\mathrm{CL}}$, becomes even more important. This has never been taken into account in the theoretical considerations. If we include the wide-band fluctuations via the CL in the numerical model [6] by an empirical coefficient K (as a multiplier to the measured value of R_d^{CL}), it allows us to fit the experimental linewidth data to the calculated one, both on the Fiske steps and on the flux flow step (see figure 3).

Since the fluctuations via the magnetic field are generated by wide-band fluctuations in the bias current, the correlation between these two noise components must be taken into account. This results in the following expression for the FFO linewidth:

$$\delta f = \left(2\pi/\Phi_0^2\right)\left(R_d^B + K * R_d^{\mathrm{CL}}\right)^2 S_i(0). \tag{4}$$

This empirical expression seems to be very similar to the formula derived recently by A Pankratov [13] on the basis of the same assumption concerning the influence of wide-band fluctuations via magnetic field.

The calculated dependence of the linewidth of the lumped tunnel junction [7, 8] in the case of wide-band fluctuations only via $I_B (K = 0)$ is shown in figure 3 by the dashed line (Curve 1), the dependence for fixed values of $R_d^{\mathrm{CL}} = 0.003\ \Omega$ and $R_d^{\mathrm{CL}} = 0.03\ \Omega$ are presented by the dotted (Curve 2) and the dash-dotted lines (Curve 3), respectively. The solid line is calculated for each experimental point taking into account all measured parameters $\left(I_B, V, R_d^{\mathrm{CL}}, \text{etc}\right)$. One can note that the calculations coincide with the measured linewidth data over the whole range of experimental parameters by using only one value of $K = 2.9$ both on the FSs and on the FFS. There is a good agreement between the calculations (4) and the experimental data plotted as a function of the bias current at two different values of the CL current (see figure 4). These values of the CL current correspond to two different regimes of FFO operation: at the FS ($I_{\mathrm{CL}} = 45$ mA) and on the FFS ($I_{\mathrm{CL}} = 50$ mA), again the

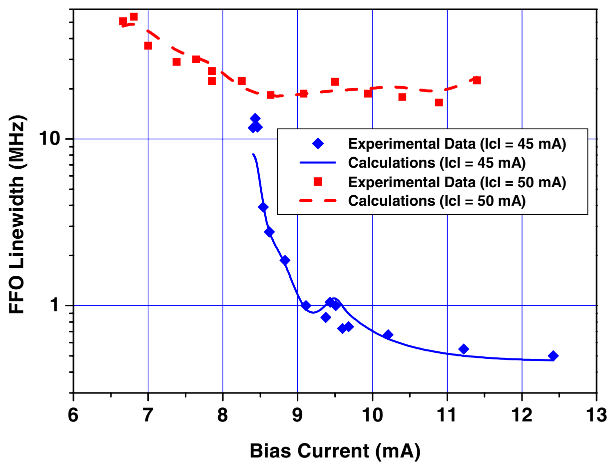

Figure 4. Dependence of the FFO radiation linewidth on the bias current for two values of the control line (CL) current (45 mA—at the Fiske step; 50 mA—on the flux-flow step). Solid lines represent the data calculated according to equation (4).

same value of $K = 2.9$ is used for all calculations. The exact value of K depends on the specific FFO design; data from figures 3 and 4 are measured for a design with the separate CL [14]. In this case the ratio between the currents, flowing on the separate CL and through the base electrode and producing the same magnetic field, is equal to 2.9. This indicates that the bias current in total adds the noise contribution via the magnetic field, but this should be studied further by special experiments, the details of these studies will be published elsewhere.

According to the theoretical considerations [7–9, 13] the radiation linewidth is determined by both the differential resistance of the FFO and the noise spectral density at low frequencies, $0 < f < \delta f_{\mathrm{AUT}}$, where δf_{AUT} is the free-running FFO linewidth. The FFO is a voltage-controlled oscillator and hence its frequency can be stabilized and the FFO linewidth can be decreased by locking to an external reference oscillator using a phase-lock loop (PLL) system with bandwidth larger than δf_{AUT}. Actually, a PLL system effectively alters both the spectral density of the external low frequency fluctuations and the differential resistances associated with the bias and control line tuning. These resistances transform current fluctuations into voltage (frequency) fluctuations according to equations (3) and (4).

For the traditional FFO design the free-running linewidth δf_{AUT} for $V > V_{\mathrm{JSC}}$ is considerably larger than the regulation bandwidth of conventional PLL systems (<1 MHz). This makes it almost impossible to phase lock a FFO on the FFS where the FFO frequency can be continuously tuned. A new FFO design [11, 14] provides a free-running FFO linewidth of about 10 MHz or smaller at all frequencies of interest. Besides a special ultra-wide-band PLL system (regulation bandwidths of about 30 MHz) has been developed. As a result, the FFO has been phase locked up to 710 GHz, these results will be published elsewhere.

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